DELVING INTO STOIC LOGIC: AN EXPLORATION

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Ι

The prevailing notion suggests that only thinkers associated with the Peripatetic school contributed significantly to the realm of ancient logic. However, this assertion appears incomplete in light of historical evidence indicating the existence of non-Peripatetic thinkers who emphasized logic and made substantial contributions. The prominence of logical inquiries beyond the confines of the Peripatetic circle is discernible from historical records. An illustrative instance recounts a regal assembly convened in ancient Alexandria wherein Diodorus Cronusⁱ, a logician, was tasked with resolving a complex logical puzzle. Despite diligent efforts, Diodorus's inability to unravel the puzzle led to despondency and ultimately, his tragic demise. Similarly, another logician, Philetus of Cosⁱⁱ, faced a comparable predicament and met a similar fateⁱⁱⁱ due to his inability to resolve the enigmatic Liar Paradox^{iv} presented before him.

When delving into the annals of ancient logic, we encounter two notable schools of thought in this field: the Peripatetic school, and the Stoic school. The founder of the Peripatetic school of logic is Aristotle and the founder of the Stoic school is Zeno^v. However, the Stoic school's evolution was steered by Chrysippus^{vi}, building upon the teachings of the Megarian philosophers. While meticulous preservation characterized the works of Aristotle and the Peripatetics, scant remnants endure from the Stoic corpus. Nonetheless, gleaned from extant sources, the intellectual depth of the Stoics underscores their scholarly significance, warranting a more comprehensive treatment in historical narratives. A pertinent inquiry arises: why did the Stoic school endure relative neglect? Diogenes Laertius^{vii}, author of *The Lives and Opinions of Eminent Philosophers*, notes the adversarial stance of Megarian philosopher Eubulides^{viii} of Miletus towards Aristotle, precipitating a longstanding feud between the Peripatetic and Megarian schools.^{ix} Given the Stoic school's foundation on Megarian principles, it became, by association, the opposition in the eyes of the Peripatetics. This protracted discord impeded the organic development of logic over centuries, inhibiting its potential progression. The Stoic school's substantive contributions to logic find validation in documented testimonials, notably by Clement^x, a Christian theologian. When

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prompted to rank eminent figures within distinct disciplines, his discernment highlighted "Homer" as the paramount poet, "Aristotle" as preeminent among scientists, and "Plato" as foremost among philosophers. However, in the realm of logicians, he notably endorsed Chrysippus, a Stoic, as the pinnacle of excellence. Clement's appraisal of Chrysippus, a Stoic logician, as the preeminent figure in logic stands out significantly. Notably, Clement's preference for Chrysippus over Aristotle in the realm of logic bears considerable weight. For nearly two thousand years, Aristotle's logic dominated the field of logic. In 1787, Immanuel Kant remarked that even then, logic hadn't progressed, leading him to suggest that it appeared complete and perfect.^{xi} Had ancient literature undergone more meticulous scrutiny, it might have disrupted the entrenched Aristotelian dominance within the realm of logic. C. S. Peirce, circa 1911, highlighted the ancient awareness of "material implication", signaling an intense historical debate on this subject. However, this line of inquiry remained largely dormant until 1927, when the eminent logician Lukasiewicz^{xii} brought to light the early Stoics' profound comprehension not only of material implication but also of various fundamental principles and methodologies integral to modern logic. Lukasiewicz's seminal work revealed the Stoics' utilization of truth-functional definitions for prevalent propositional connectives. Notably, he underscored their nuanced differentiation between arguments and corresponding conditional propositions. Most significantly, the Stoics pioneered the development of an inference-schema calculus, establishing five foundational valid inference-schemas or valid argument forms which have been discussed later in section II. Peripatetic commentators, including Ammonius, Simplicius, Prantl, Zeller, and others, have been noted for their misinterpretations of several Stoic concepts. One such instance involves the Stoic concept of "Lekta", defined as "meaning". Regrettably, certain Peripatetic commentators have either deliberately or inadvertently misconstrued this concept, deviating from its original Stoic understanding.

...however clear these distinctions may originally have been, much confusion surrounded them in later centuries, especially in the minds of the Aristotelian commentators. For example, Ammonius says that the Lekton of the Stoics is an intermediate entity between the thought and the thing, but in Simplicius we read that Lekta and thoughts are identical....^{xiii}

Another illustration of this phenomenon lies in Prantl's citation of appropriate sources to expound upon the Stoic distinction between defective and complete Lekta. However, his misunderstanding surfaced when he conflated the concept of Lekta with the Stoic notion of assertible, leading to a comprehensive misinterpretation of the underlying principles.

According to Prantl, "The Stoics divided propositions into deficient and complete" and in support of this he cites passages in which Sextus and Diogenes say that Lekta are so divided. This

indicates that Prantl did not understand the Stoic definition of a proposition as a "complete Lekton, assertoric in itself".^{xiv}

According to these commentators, the Stoic concept of arguments and conditionals too are identical. But nothing in the Stoic writings justifies this. There are many more such examples. After observing these inappropriate portrayals of Stoic logic, Lukasiewicz concluded that the history of logic should be rewritten.

The provenance of Stoic logic confronts a challenge rooted in the temporal discrepancy between the available Stoic writings and their thematic focus. The extant Stoic texts predominantly hail from the later period, emphasizing ethical discourse over logical intricacies. Contrarily, the earlier Stoics were prolific in their exploration and documentation of various logical domains. Regrettably, the absence of surviving primary sources dedicated to Stoic logic necessitates reliance on secondary accounts. Sextus Empiricus, a renowned skeptic philosopher from the third century A.D., emerges as a crucial figure in elucidating Stoic logical tenets. His comprehensive expositions on Stoic logic predominantly reside within the volumes of Book II (*Against the Rhetoricians*) and Book VIII (*Against the Logicians*) of the *Against the Mathematicians* series. Additionally, Diogenes Laertius, the author of *Lives of Eminent Philosophers*, penned in the fourth century B.C., provides glimpses into the landscape of Stoic logic.

Π

As per Stoic philosophy, logic constitutes one facet of the broader philosophical triad, alongside physics and ethics.^{xv} For them, logic is very important for understanding physical world properly and also it is a necessary component of moral life. According to the Stoics, the goal of physics is to comprehend worldly phenomena, examine them, and provide explanations, while ethics aims to harmonize human life with the natural order. Conversely, logic focuses on discerning truth from falsehood, enabling the discovery of truths within the domains of reality that pertain to other branches of philosophy. Therefore, logic maintains a close relationship with other philosophical components, prompting the Stoics to consider it a crucial element of philosophy. The Stoics' valuation of logic's significance within the triad of philosophical pursuits becomes apparent through the analogies they employed. They compared philosophy to a fruit-filled garden, where the physical aspect corresponds to the height of the plants, the ethical aspect mirrors the sweetness of the fruits, and the logical aspect embodies the durability of the walls.^{xvi} Additionally, in their comparison of philosophy to an egg, ethics nourishes the yolk, physics sustains the egg white, and logic assumes the protective role of the egg's outer shell.^{xvii} Similarly, comparing philosophy to an animal, they equate the physical part to blood and flesh, the logical part to bones and sinews, and the ethical part to the soul.^{xviii} The significance of walls in a garden, the

importance of the outer shell of an egg, and the crucial role of bones and sinews in an animal are universally recognized. Garden walls ensure structural integrity and protection, preserving and prolonging the fruits within. Eggshells offer essential defense and structural support, safeguarding the delicate contents. Similarly, bones and sinews provide the framework and support essential for an animal's movement, stability, and overall bodily function. The primary objective of Stoic logic lies in facilitating clear and accurate thinking while safeguarding against fallacious reasoning prevalent in various forms of rational discourse. Essentially, logic serves as a tool to guide individuals in conducting discussions and debates with precision, addressing inquiries methodically, comprehensively exploring arguments both in favour of and against a given proposition, distinguishing truth from falsity, elucidating ambiguous statements, and resolving paradoxes. Generally, logic aims to establish a genuine and enduring comprehension of the world-an understanding deemed vital for humans to lead a life characterized by sound reasoning. Zeno, the founder of the school, segmented logic into two distinct disciplines: rhetoric and dialectic.xix Rhetoric denotes the skillful art of articulate expression through cohesive, uninterrupted discourse, while dialectic involves the adept orchestration of dialogues through concise question-and-answer exchanges.^{xx} Within dialectic lies the concept of "Lekta", defined by Stoics as "meaning". Lekta is further divided into two categories: deficient and complete. Deficient Lekta refers to expressions where the enunciation remains incomplete, such as "writes", prompting the query "Who?" On the other hand, complete Lekta denotes self-sufficient enunciations, like "Socrates writes". xxi Under complete Lekta, Stoics included exclamations, questions, imperatives, orders, and assertibles etc.^{xxii} However, only assertibles hold relevance in Stoic logic due to their truth functionality, capable of being either true or false. As per the Stoics, an argument comprises a compilation of such assertibles, with certain assertibles designated as premises and an assertible as the conclusion. Diogenes Laertius, in his renowned work Lives of Eminent Philosophers, said:

An argument, according to the followers of Crinis, consists of a major premiss, a minor premiss, and a conclusion, such as for example this: "If it is day, it is light; but it is day, therefore it is light". Here the sentence "If it is day, it is light" is the major premiss, the clause "it is day" is the minor premiss, and "therefore it is light" is the conclusion. A mood is a sort of outline of an argument, like the following: "If the first, then the second; but the first is, therefore the second is".^{xxiii}

Similar to the valid argument forms acknowledged in Aristotelian logic and propositional logic, valid argument forms were also recognized in Stoic logic. Stoic logicians coined the term "Indemonstrable" for those elementary valid argument forms. Indemonstrables, according to Stoics, are those that do not

require demonstration or proof.^{xxiv} According to Diogenes' *Lives of Eminent Philosophers*, Stoics accepted five types of indemonstrables.

The first type of indemonstrable adheres to the following form:

"If the first, then the second

The first

Therefore, the second"xxv

For instance:

"If it is day, it is light

It is day

Therefore, it is light"

This form orchestrates a conditional statement as the major premise, utilizing the antecedent of the conditional as the minor premise and the consequent as the conclusion.^{xxvi}

Moving to the second type of indemonstrable, it follows this pattern:

"If the first, then the second

It is not that: the second

Therefore, it is not that: the first"

For instance:

"If it is day, it is light

It is not that: it is light

Therefore, it is not that: it is day"

This type constructs its argument around a conditional statement as the major premise, employing the contradiction of the consequent of the conditional as the minor premise and the contradiction of the antecedent as the conclusion.^{xxvii}

The third type of indemonstrable operates under this structure:

"It is not that: (the first and the second)

The first

Therefore, it is not that: the second"

For instance:

"It is not that: (Aristotle is dead and Aristotle is alive)

Aristotle is dead

Therefore, it is not that: Aristotle is alive"

This category employs the contradiction of a conjunctive statement as the major premise, one of the conjuncts of the statement as the minor premise, and the contradiction of the other conjunct as the conclusion.^{xxviii}

The fourth type of indemonstrable follows this framework:

"The first or the second

The first

Therefore, it is not that: the second"

For instance:

"Either it is day or it is night

It is day

Therefore, it is not that: it is night"

This form comprises a disjunctive statement as the major premise, utilizing one of the disjuncts of the statement as the minor premise and the contradiction of the other disjunct as the conclusion.^{xxix}

Lastly, the fifth type of indemonstrable adheres to this structure:

"The first or the second It is not that: The first

Therefore, the second"

For instance:

"Either it is day or it is night

Not: It is day

Therefore, it is night"

In this configuration, a disjunctive statement serves as the major premise, contradicting one of its disjuncts as the minor premise, and affirming the other disjunct as the conclusion.^{xxx}

III

In logic, the form of an argument is closely related to its validity. Numerous arguments can be constructed from an argument form. Logicians provide us with principles and techniques that allow us to determine which arguments are valid and which are invalid. The task of a logician is not to examine all possible arguments and determine whether they are valid. However, a logician can identify which argument forms are valid. If an argument form is valid, then every argument adhering to that form will be valid. Arguments that fit none of the valid argument forms are invalid. Therefore, it is crucial to determine the form of an argument in every logical framework. In determining the form of an argument in Stoic logic, the simple propositions that comprise an argument are symbolized.

Let it be understood through an instance.

If it's day, then it's light.

It's day.

Therefore, it's light.

This argument consists of three propositions. Form of this argument for Stoics is:

If the first, then the second.

The first.

Therefore, the second.

Here in this argument the major premise serves as a non-simple proposition and to determine the form, the constituent simple propositions "It's day" and "It's light" are symbolized by the substitution of "The first" and "The second" respectively. Since an argument form for Stoics is determined by the propositions composing the argument, Stoic logic can be called propositional logic.

Here it's important to note that the Stoic differentiation between deficient and complete Lekta bears resemblance to Gottlob Frege's distinction between incomplete and complete expressions. According to Frege's analysis, sentences contain both complete and incomplete expressions, both of which contribute to the overall structure of a sentence. Incomplete expressions, as per Frege's analysis, are integral components within a complete sentence. The defining quality of an incomplete expression is its

requirement for saturation or completion.^{xxxi} This distinction can be elucidated through an example: consider the sentence "Bankim Chandra Chattopadhyay is the author of Kapalkundala". In Frege's view, "Bankim Chandra Chattopadhyay" stands as a complete expression, being a proper name, while "the author of Kapalkundala" is also deemed a complete expression, being a definite description. Beyond these complete expressions, there exists another form derived from the sentence: "______is the author of _____". According to Frege, such expressions are classified as incomplete. The hallmark of an incomplete expression lies in its inherent need for saturation or completion. Only when this incomplete portion is filled in does it result in a complete sentence.

Additionally, it's important to note that the Stoic notion of assertibles aligns with the concept of propositions. Both assertibles and propositions share common traits: they are expressed through complete declarative sentences and possess truth functionality. Sentences encompass diverse types like exclamations, questions, imperatives, orders, and propositions, yet only propositions hold truth functionality. Similarly, among the different types of complete Lekta accepted in Stoic logic, only assertibles possess truth functionality. Consequently, the Stoic concept of assertibles can be linked back to the concept of propositions. This leads to an important query: are Stoic assertibles and the concept of propositions indistinguishable? Diogenes Laertius's *Lives of Eminent Philosophers* presents an insightful observation in this regard:

The Greek word for judgement $(\dot{a}\xi i\omega\mu\alpha)$ is derived from the verb $\dot{a}\xi io\tilde{v}v$, as signifying acceptance or rejection; for when you say "It is day," you seem to accept the fact that it is day. Now, if it really is day, the judgement before us is true, but if not, it is false.^{xxxii}

In general discourse, propositions and assertibles appear synonymous. However, upon closer examination, the aforementioned reference delineates their divergence. An assertible, such as "It is day", holds truth when it corresponds to the current state of affairs—truth at daylight and falsehood at night, thus demonstrating the Stoic perspective that the truth assessment of an assertible is contingent upon temporality. This intricate distinction can be further elucidated through an illustrative instance. Consider the assertible "Sachin is an international cricketer", perceived as false in Stoic logic due to Sachin's retirement from international cricket. However, in the era when Sachin actively played international cricket, this assertible would have been deemed true. In contrast, the determination of truth value in a proposition, unlike the Stoic concept of assertible, remains unaffected by temporality. A proposition's truth status remains unchanging; it is either perpetually true or false. Consequently, within modern logic, the proposition "Sachin is a cricketer" is considered consistently true due to its timeless nature.

In the context five types of indemonstrables, it becomes evident that the form of the first, second, and fifth type of indemonstrable accepted by the Stoics aligns with the elementary valid argument forms recognized in propositional logic, namely Modus Ponens, Modus Tollens, and Disjunctive Syllogism. By applying certain rules of replacement, the third type of indemonstrable can also be transformed into Disjunctive Syllogism. The form of the third type of indemonstrable is as follows:

"It is not that: (the first and the second)

The first

Therefore, it is not that: The second"

Here the major premise 'It is not that: (the first and the second)' can be symbolised as

 \sim (The first . The second)

Which by means of De Morgan's law can be transformed into

 \sim The first $v\sim$ The second

The minor premise 'The first' by means of rule of Double Negation can be transformed into

 $\sim \sim$ The first

Now this indemonstrable can be rewritten as

"~ The first v ~ The second

 $\sim \sim$ The first

Therefore, ~ The second"

which is none other than the form of Disjunctive Syllogism. This is how the third type of indemonstrable, as upheld by Stoics, can be transformed into Disjunctive Syllogism.

However, within propositional logic, the form of the fourth type of indemonstrable cannot be recognized as a valid argument form.

Upon scrutinizing the five types of indemonstrables advocated by Stoics, an observation emerges: a Stoic argument contains a non-simple assertible composed of simple assertibles, truth-functionally combined to generate the non-simple assertible. Similarly, within propositional logic, an argument generally involves at least one truth-functional compound proposition formed through the amalgamation of truth-functional simple propositions.

IV

From the preceding discussion, it's evident that in ancient times, the Stoics significantly amplified their contributions to the domain of logic. However, it remains disheartening that even today, they fail to receive the recognition they truly merit.

Propositional logic, cultivated under the guidance of eminent philosophers such as Gottlob Frege and Bertrand Russell, marked an epoch of unparalleled eminence. In both propositional and Stoic logic, the form of an argument is derived by symbolizing the propositions that comprise an argument. Upon observation, it becomes apparent that the Stoic delineation between deficient and complete Lekta shares similarities with Gottlob Frege's differentiation between incomplete and complete expressions. Furthermore, while the Stoic notion of assertibles aligns with the concept of propositions, discernible disparities exist between the two. Notably, the truth value of an assertible is contingent upon temporality, unlike that of a proposition. Additionally, the first, second, and fifth types of indemonstrables embraced by the Stoics align with elementary valid argument forms recognized in propositional logic: Modus Ponens, Modus Tollens, and Disjunctive Syllogism. Moreover, applying specific rules of replacement enables the transformation of the third type of indemonstrable into Disjunctive Syllogism. Upon examining the five types of indemonstrables advocated by Stoics, it becomes apparent that a Stoic argument comprises a non-simple assertible formed truth-functionally by combining simple assertibles. Similarly, within propositional logic, arguments commonly involve at least one truth-functional compound proposition derived from the combination of truth-functional simple propositions. Upon acknowledging these resemblances, one can conclude that traces of propositional logic are evident in Stoic logic. In the present era, propositional logic has reached a commendable level of eminence. Yet, neglecting the potential impact of Stoic logic on the fundamental tenets of propositional logic would be amiss. Indeed, the cornerstone of propositional logic may have been laid by the Stoics in antiquity.

Notes and References:

ⁱ According to Stanford Encyclopedia of Philosophy, Diodorus was a pioneering logician, and the most celebrated member of the Dialectical School of the 4th-3rd c. BCE. His contributions to logic—in particular, definitions of modal terms and the criteria for a sound conditional—are covered in the article on the Dialectical School.

ⁱⁱ According to Britannica, Philitas of Cos, Philitas also spelled Philetas, (born *c*. 340 BC, Cos [Aegean Islands, now in Greece]—died *c*. 270 BC), Greek poet and grammarian, regarded as the founder of the Hellenistic school of poetry, which flourished in Alexandria after about 323 BC. He is reputed to have been the tutor of Ptolemy II and the poet Theocritus.

ⁱⁱⁱ Philetus wrote the following before he took his own life:

Philetus of Cos am I

'twas The Liar who made me die

And the bad nights caused thereby

^{iv} According to Britannica, **Liar paradox**, <u>paradox derived</u> from the statement attributed to the Cretan prophet <u>Epimenides</u> (6th century BCE) that all Cretans are liars. If Epimenides' statement is taken to imply that all statements made by Cretans are false, then, since Epimenides was a Cretan, his statement is false (i.e., not all Cretans are liars). The paradox in its simplest form arises from considering the sentence "This sentence is false." If the sentence is true, then it is false, and if it is false, then it is true. ^v According to Britannica, **Zeno of Citium**, (born *c*. 335 BCE, <u>Citium</u>, Cyprus—died *c*. 263, Athens), Hellenistic thinker who founded the <u>Stoic</u> school of <u>philosophy</u>, which influenced the development of philosophical and <u>ethical thought</u> in Hellenistic and Roman times. He went to <u>Athens</u> about 312 BCE and attended lectures by the Cynic philosophers <u>Crates of Thebes</u> and <u>Stilpon</u> of Megara, in addition to lectures at the Academy. Arriving at his own philosophy, he began to teach in the Stoa Poikile (Painted Colonnade), whence the name of his philosophy. Zeno's philosophical system included logic and theory of knowledge, physics, and ethics—the latter being central.

^{vi} According to Internet Encyclopedia of Philosophy, Chrysippus was among the most influential philosophers of the <u>Hellenistic period</u>. He is usually thought of as the most important influence on Stoicism. A later Stoic catchphrase ran, "If Chrysippus had not existed, neither would the Stoa."

(*Lives* 292). ... His work in logic was considerable, as he developed, as an alternative to the logic of Aristotle, a kind of propositional logic.

^{vii} According to New World Encyclopedia, Diogenes Laertius (c. 200 - 250 C.E.) was an early doxographer who compiled biographies of ancient Greek philosophers in his seminal work, *Lives of Eminent Philosophers*. The ten books contain quotations and anecdotes from the lives of nearly one hundred philosophers, including 45 important figures, from Thales (585 B.C.E.) to the Skeptics of the late second century C.E.

^{viii} According to Britannica, **Eubulides Of Miletus**, (born 4th century BC), a member of the <u>Megarian</u> <u>school</u> of <u>philosophy</u> in Athens and renowned as an inventor of logical <u>paradoxes</u>, the most famous of which is "The Liar" ("Does a man who says that he is now lying, speak truly?"). He was a contemporary of <u>Aristotle</u>, whom he attacked, and tradition says that he was a teacher of <u>Demosthenes</u>.

ix Hicks (Ed.). Diogenes Laertius: Lives of Eminent Philosophers. Harvard University Press,

1925. Book II, section 109

^x According to Britannica, **St. Clement of Alexandria**, (born 150 CE, Athens—died between 211 and 215; Western feast day November 23; Eastern feast day November 24), Christian <u>Apologist</u>, missionary theologian to the Hellenistic (Greek cultural) world, and second known leader and teacher of the catechetical <u>School of Alexandria</u>. The most important of his surviving works is a trilogy <u>comprising the *Protreptikos*</u> ("Exhortation"), the *Paidagōgos* ("Instructor"), and the *Strōmateis* ("Miscellanies").

^{xi} Muller, Max, translator. *Critique of Pure Reason*. Second Edition, New York, Macmillan, 1925. p 688. ^{xii} According Stanford Encyclopedia of Philosophy, Jan Łukasiewicz (1878–1956) was a Polish logician and philosopher who introduced mathematical logic into Poland, became the earliest founder of the Warsaw school of logic, and one of the principal architects and teachers of that school. His most famous achievement was to give the first rigorous formulation of many-valued logic. He introduced many improvements in propositional logic, and became the first historian of logic to treat the subject's history from the standpoint of modern formal logic.

xiii Mates, B. Stoic Logic. University of California Press, 1961. p. 12

^{xiv} Ibid. p. 88

xv Hicks (Ed.). Diogenes Laertius: Lives of Eminent Philosophers. Harvard University Press,

1925. Book VII, section 39

^{xvi} Ibid. section 40

^{xvii} Ibid

^{xviii} Ibid

xix Ibid. section 41

^{xx} Ibid. section 42

^{xxi} Ibid. section 63

^{xxii} Bett, R. (Ed.). *Sextus Empiricus: Against the Logicians (Cambridge Texts in the History of Philosophy)*. Cambridge University Press, 2005. section 71-73

^{xxiii} Hicks (Ed.). *Diogenes Laertius: Lives of Eminent Philosophers*. Harvard University Press, 1925. Book VII, section 76

xxiv Ibid. section 79

^{xxv} Unlike Peripatetic logicians, ordinal numbers were used as variables by the Stoics in place of entire propositions for disclosing the form of an argument.

^{xxvi} Hicks (Ed.). *Diogenes Laertius: Lives of Eminent Philosophers*. Harvard University Press, 1925. Book VII, section 80

^{xxvii} Ibid.

^{xxviii} Ibid.

^{xxix} Ibid. section 81

^{xxx} Ibid.

^{xxxi} Frege, Gottlob. "Function and Concept." *The Frege Reader*, edited by Michael Beaney, Blackwell Publishing, 2009, pp. 133.

^{xxxii} Hicks (Ed.). *Diogenes Laertius: Lives of Eminent Philosophers*. Harvard University Press,1925. Book VII, section 65